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Demographic Factors Affecting Unfunded Pensions

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Abstract

Population aging affects more and more countries. An important aspect of population aging is the role of public pension systems to ensure that retired workers have adequate pension to support themselves. There are many types of pension systems. This paper examines the type found in most countries, the unfunded pension system, also known as pay-as-you go old-age insurance. This paper examines the demography of the unfunded pension system for birth cohorts in the United States using U.S. census data and simulations from population projections. The paper analyses the implicit rate of return for birth cohorts over time. Results show that cohorts born during the baby boom years, 1946 to 1967, and earlier years show a positive return. However, cohorts born after 2000 have returns that are negative. Declines in pay-as-you-go old-age insurance returns are typical for unfunded pensions as they mature, and negative returns are usual for populations when the birth rate decreases under fixed economic conditions. The return of unfunded pensions can be kept positive by greatly increasing fertility, a demographic change that is unlikely in the United States and many countries that are experiencing population aging. The conclusions of this paper for the United States are broadly applicable to other countries with unfunded pension systems that are experiencing or have experienced a transition from high fertility to replacement or sub-replacement fertility.

Introduction

Unfunded or pay-as-you-go pension systems pay out to current beneficiaries based on current contributions (Auerbach and Lee, 2011; Bloom and Canning, 2004). Such systems are heavily dependent on the ratio of persons in the beneficiary ages to those in the contributing ages (Ludwig and Reiter, 2010). If fertility is relatively high and mortality is comparatively low, the population will be young and rapidly growing and the ratio of beneficiaries to contributors will be low so that the scheme will seem inexpensive and reasonable (Lee, 2003). An unfunded system will also appear to be inexpensive in a rapidly expanding economy. If the population and the economy are stationary, then, an unfunded scheme is more expensive. But the threat of a stationary population (a population with replacement-level fertility and zero population growth) is only part of the reason for concern with unfunded pension schemes (MacKellar, 2000; Thøgersen, 1998).

There is a worse condition than a stationary population that is an immediate concern. In the United States, large birth cohorts of late 1940s, 1950s, and early 1960s began to retire in the second decade of the twentieth-first century, and the number of contributors to the pension systems are smaller because of low fertility since 1970s (Börsch-Supan, 2004; Brooks, 2002 Modigliani, Ceprini, and Muralidhar, 1999). Among larger high-income countries, previous replacement fertility in France, United Kingdom, and the United States have decreased to sub-replacement levels with total fertility rates ranging from 1.7 to 1.9 children per woman (United Nations, 2020). Population is decreasing in more than a dozen countries at present, and the ratio of persons in working ages to older ages is falling below that existing in stationary populations. This worse-than-stationary population ratio will persist as the post-war baby boom passes through retirement ages.

Although the numbers presented in this paper are for the United States, the general demographic outcome will apply for any low-fertility population that previously experienced higher fertility. The lessons about unfunded pension systems, for example, will help to understand better the evolving situation in several Asian populations experiencing fertility decline. Based on U.N. Population Division (2020) analysis, current average total fertility rate of 1.7, 2.2 and 2.4 for East Asia, Southeast Asia, and South Asia respectively, are projected to decrease to replacement or sub-replacement levels ranging from 1.7, 2.0, and 2.1 in 2030. Lower future fertility in more populous countries of Bangladesh, China, India, Indonesia, and Japan and in many countries with smaller populations will lead to unfunded pension outcomes similar to those described in this paper.

The effect of economic growth on unfunded pensions is similar to that of population growth. An increase or decrease of one percent economic growth will have the same effect (Geanakoplos, Magill, and Quinzii, 1999; Krueger and Ludwig, 2007). But there is an important difference between them. An economy can have much higher and lower rate of growth than a population. For modern industrialized countries, annual population growth rate typically varies between negative and positive one percent. Economic growth has been more rapid than population growth in recent decades and has exceeded population growth in all but a few years (Mankiw and Weil, 1989; Poterba, 2004). If economic growth is moderate or slow, then the demographic situation can have an important influence on unfunded pensions.

The purpose of this paper is to describe the effect of demographic factors on unfunded pensions. We ignore economic factors, such as economic growth, that influence unfunded pensions. Rather, demographic factors are considered as the sole source of change on unfunded pensions over the lifetime for successive birth cohorts. For consideration of major factors affecting the U.S. public pension system, see the U.S. Social Security Administration's (2020) recent annual report that offers forecasts of the contributions and payments for future decades. Rather than forecasts over time, the simulations in this paper describe net transfers between generations over entire lifetime. Unfunded pension schemes are too often viewed in cross-sectional terms based on the current calendar year (Barr and Diamond, 2008; Diamond and Orszag, 2005; Lindbeck and Persson, 2003). Such a short-term view does not offer an adequate description of the demography of these schemes (Samuelson, 1958). Long-term stability depends to some extent on a degree of equity between generations, and this requires analysis of birth cohorts over several generations.

This paper asks whether a population that is growing faster or slower influences unfunded pensions for successive generations. If there is higher or lower fertility, or higher or lower mortality, what difference does it make? If there is more or less immigration, what effect does that have? This paper updates an earlier publication by Keyfitz (1985) with two contributions: (1) estimates of the implicit rate of return for unfunded pensions for the 1830-34 to 2040-44 U.S. birth cohorts and (2) simulations of the effect of fertility, mortality, and immigration based on more recent data. This paper owes its conceptual foundation to Keyfitz's (1985) paper.

Unfunded Pensions

Measure of Return

There are several possible measures for the comparison of costs and benefits of an unfunded pension scheme (Fenge and Werding, 2003). Total contributions to the scheme for an individual could be a comparison to total benefits to obtain the absolute profit or loss for that person. Or the same comparison could be made for a birth cohort of persons. The ratio of benefits to costs to a person offers another measure, indicating how many dollars a person receives in return for each dollar contributed.

The measure used in this paper is the implicit rate of return, which is calculated as the rate of interest that makes contributions equal to benefits when both are discounted back to birth (see Knell, 2010a; Knell 2010b; and Knell, 2013 for extensive discussion of economic aspects of implicit rate of return). This is the way that most private funded pensions are evaluated, and it is a common measure for bonds, real estate, and other investments. The implicit rate of return offers a reasonable comparison of different rules, such as beginning a pension at age 60, 65, or 70 years, for example, or for different situations, such as high or low fertility, or high or low net immigration.

To assess the return of an unfunded pension scheme, the characteristics of the operation need to be described. Suppose each worker is promised a fixed sum, say $10,000, for each year the worker is alive after age β (say, 65 years), and in return, during ages between α and β (say, 20 to 64 years) the worker is to bear equal share each year of the cost of providing the same benefit to the old people who are alive. In essence, this is the way in which most unfunded pension schemes are conceived, which is called a *defined benefit scheme* because workers are promised a specific benefit when they retire. In practice, not all workers contribute to an unfunded pension and the benefit is not equal to the wage but usually replaces only a fraction of it. These differences are disregarded in calculations here and are described for conceptual clarity. If only 60 percent of a salary is replaced, then this part is related to the contribution, and all the following calculations are applicable. The tables below compare various conditions, maintained for long time periods, for a conventional fixed pension of, say, $10,000 per year.

Suppose an individual goes through their working life making contributions to the support of old people alive each year. Each calendar year's contribution is taken to be equal to the ratio of old people to working people of that year. In this approach, this is the cost to the payer as they pass through working life of that $10,000 per year expected during retirement. Stated formally, the number of beneficiaries in year t is

$$\int_{\beta}^{\omega} p(x,t)dx$$

where β is the age of retirement, ω is the highest age that anyone lives, and $p(x,t)dx$ is the number of persons in the population between age x and x+dx at time t. The number of contributors in the year t is

$$\int_{\alpha}^{\beta} p(x,t)dx$$

where α is the age of starting work and β is the age of retirement. For such a defined benefit scheme, the premium paid each year by everyone between α and β is the ratio of the number of beneficiaries to the number of contributors. The ratio for the premium at time *t* is

$$Prem(t) = \int_{\beta}^{\omega} p(x,t)dx / \int_{\alpha}^{\beta} p(x,t)dx$$

The equation is to be solved for *r*, the implicit rate of return, consists of the expected payment of Prem(t) against the expected benefit of unity per year, or

$$\int_{\beta}^{\omega} e^{-rx}\, l(x)dx = \int_{\alpha}^{\beta} e^{-rx}\, l(x)Prem(c + x - 20)dx \qquad (1)$$

where *c* is the calendar year when the cohort starts working (assumed here to be 20 years). Call the left-hand side A and the right-hand side B, and we then need to find a value of *r* in which A=B. The implicit rate of return is quickly computed by functional iteration. Calculate the discounted benefit (A) and the discounted payment (B), with both discounted to birth for calculation convenience, using an initial trial value for the rate of return. In practice, convergence is reached to six significant digits in five or fewer iterations.

Results would not be identical with an opposite approach that is less conventional: fix the contributions at, say, $10,000 per year and divide the total proceeds among those who are drawing on the pension scheme, which is called a *defined contribution scheme* because workers make a specific contribution and receive a variable amount after retirement which depends upon what workers have contributed. Both the pattern and the inequalities between birth cohorts would be quite different in the defined benefit and the defined contribution schemes. This paper devotes attention to a defined benefit scheme, which is the conventional public pension scheme found in the United States and in most countries. The last section of the paper includes brief discussion of differences with a defined contribution pension scheme that holds the contributions constant.

For a defined contribution scheme, the contribution received each year by everyone aged β to ω is the ratio of the number of contributors to the number of beneficiaries, where the benefits at time t is

$$Ben(t) = \int_{\alpha}^{\beta} p(x,t)dx \Big/ \int_{\beta}^{\omega} p(x,t)dx$$

The equation to be solved for the implicit rate of return is

$$\int_{\alpha}^{\beta} e^{-rx}\, l(x)dx = \int_{\beta}^{\omega} e^{-rx}\, l(x)Ben(f + x - 65)dx \qquad (2)$$

where f is the year when the cohort is 65 years old. Equation (2) is also solved by functional iteration.

Trends in the Rate of Return

We calculate the implicit rate of return for 1830-34 to 2040-44 birth cohorts for the U.S. population. This requires age data from 1850 (when the 1830 birth cohort reached 20 years of age) to 2130 (when the 2040 birth cohort reaches the end of life). Age data are from two sources: (1) tabulations of microdata samples from U.S. censuses from 1850 to 2010, which are interpolated for birth cohorts for the five-year periods between decennial censuses and (2) U.S. Census Bureau (2017) population projections for 2017 to 2060. For projections for 2060 to 2130, this paper makes separate population projections if mortality, fertility, and net immigration prevailing in 2060 continues unchanged until 2130.

The implicit rate of return based on these age data is shown as the solid green line, labelled "demographic *r*", in Figure 1 for 1830-34 to 2040-44 birth cohorts. The annual rate of return is 2.09 percent for the 1830-34 birth cohort, when relatively high fertility produced a young age structure. Steady fertility declines in subsequent years results in an increasingly older population, which declines the implicit rate of return to 0.95 percent for the 1920 birth cohort, below zero (-0.05 percent) for the 1995-99 birth cohort and continuing with negative rates of return for subsequent birth cohorts.

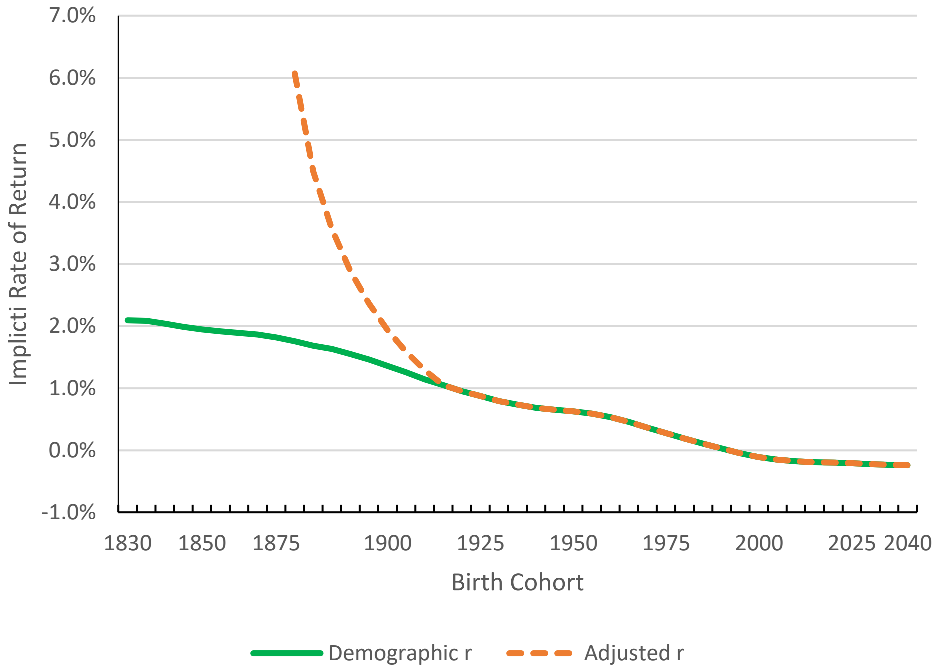

Figure 1: Trend in the rate of return, 1830-2040.
Source: Author

The trend in the rate of return can be modified (called the "adjusted *r*") to illustrate the fact that the U.S. public pension system began in 1935, with contributors beginning in the second half of the 1930s but offering benefits to all elderly persons (shown as the dotted orange line in Figure 1). This meant that birth cohorts born before 1875 received benefits having not made contributions prior to 1935. The 1875-79 birth cohort began to receive benefits at age 65 in 1940 having made contributions only for the previous five years. The 1875-79 birth cohort obtained an implicit annual rate of return of 6.1 percent. Earlier birth cohorts (born before 1875) received an infinite rate of return because they did not make contributions. The 1920-24 birth cohort is the first birth cohort that made contributions throughout their young adult years and had the same implicit "demographic" and "adjusted" rates of return.

Figure 2 displays the implicit rate of return for the baby boom and subsequent generations for 1945-49 to 2040-44 birth cohorts. The important point is that there is a positive rate of return for the 1945 to 1969 birth cohorts, those born during the baby boom. The subsequent generation – those born during 1970 to 2000 – have a declining rate of return that becomes negative for those born after 1990. Births occurring after 2000 have decreasingly negative returns.

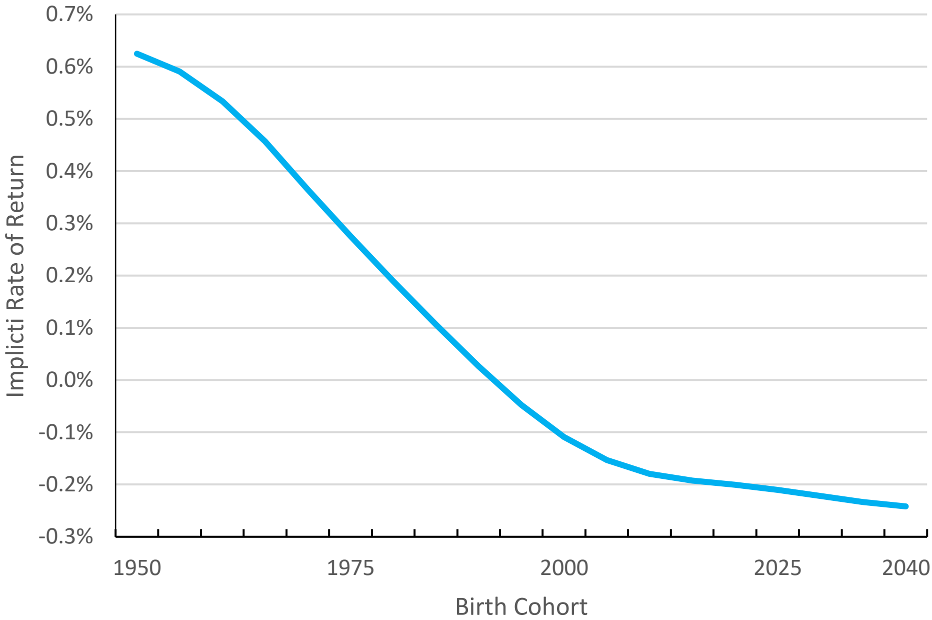

Figure 2: Implicit rate of return for baby boom and subsequent generations 1945-2044
Source: Author

The next section describes simulations that examine the effects of variations in fertility, mortality, and immigration on the implicit rate of return for the 1990-1994 to 2080-84 birth cohorts.

Simulation

Simulation for the effect of fertility, mortality, and immigration on the implicit rate of return uses projections from 2010 to 2170. Simulation starts with 2010 U.S. population, by age and sex, with results reported for both sexes combined. The population is projected 160 years into the future, up to 2170, with varying assumptions about fertility, mortality, and net immigration. Initially, fertility, mortality, and immigration are fixed to provide baseline comparisons for other simulations. For the baseline, total fertility rate is held constant at 2.00, life expectancy at birth is 76.4 years for males and 81.2 years for females, and net immigration of 1,000,000 per year, with age and sex distribution of immigrants during 2010-2015. We can judge result of these assumptions by examining the effect of higher or lower fertility, mortality, and net immigration respectively. As described below, we can use different combinations of assumed fertility, mortality, and net immigration to describe how demographic factors affect the rate of return for unfunded pensions.

Table 1: Population projection for United States, 2010 to 2170
(Population in 1,000s for both sexes combined)

Age	Year								
	2010	2030	2050	2070	2090	2110	2130	2050	2170
All	308746	357296	383718	404251	421872	430399	447478	455995	464275
0-4	20201	21663	22814	23784	24785	25763	26705	27623	28516
5- 9	20349	22052	23019	24002	25032	26009	26471	26922	27360
10-14	20677	22187	22972	24049	25087	26059	26525	26980	27420
15-19	22040	22060	22930	24103	25108	26083	26555	27012	27455
20-24	21586	21765	23216	24360	25322	26317	26792	27250	27696
25-29	21102	22050	23736	24693	25666	26685	27159	27619	28067
30-34	19962	22655	24144	24917	25978	27001	27472	27935	28385
35-39	20180	24091	24111	24964	26115	27102	27573	28039	28491
40-44	20891	23328	23503	24919	26037	26976	27455	27923	28374
45-49	22709	22216	23131	24764	25691	26633	27122	27585	28034
50-54	22298	20324	22890	24310	25048	26061	26546	27001	27448
55-59	19665	19735	23376	23396	24192	25265	25727	26174	26617
60-64	16818	19517	21709	21867	23144	24152	24581	25025	25458
65-69	12435	20012	19582	20362	21762	22558	22973	23408	23821
70-74	9278	18145	16566	18596	19726	20313	20734	21140	21520
75-79	7318	14127	14169	16706	16723	17282	17687	18034	18371
80-84	5743	9786	11319	12552	12641	13366	13686	13956	14234
85+	5493	11583	20530	21907	23814	25074	25714	26370	27007

Note: The population projection applies 2010 age-specific mortality rates for males and females, 2010 age- specific fertility rates, and annual immigration of 1,000,000 persons distributed by age and sex like recently arrived foreign-born persons in the 2015 American Community Survey.
Source: Author

The initial baseline projection shows a population of 394.1 million by 2060, compared to a figure of 420.3 million in the 2017 medium-level projection published by the U.S. Census Bureau (2018) and a figure of 412.4 million in the 2019 medium-variant projection of United Nations (2019). The differences are mainly due to fertility, mortality, and immigration being fixed in the baseline simulation, while the U.S. Census and United Nations projections assume mortality improvement, rising net immigration, and slight decline in total fertility rate.

Fertility

Based on U.S. fertility data by age, we use the following age-specific fertility rates per 1,000 women of reproductive age: 74.5 for 15-19, 107.3 for 20-24, 100.6 for 25-29, 74.5 for 30-34, 37.2 for 35-39, and 6.0 for 40-44 age groups – which corresponds to a total fertility rate of 2.00. We examine rates for five fractions 0.50, 0.75, 1.00, 1.25, and 1.50 times these age-specific fertility rates, which correspond to total fertility rates of 1.00, 1.50, 2.00, 2.50, and 3.00 respectively.

Mortality

We create life tables for different levels of life expectancy at birth (e_0) and life expectancy at age 65 (e_{65}) using Brass relational life table model. We use 2008 U.S. life tables (Arias, 2012) as the standard for creating other life tables, where the number surviving to age x, called l_x^s, is referred to as the standard life table below.

The baseline model for simulation uses an e_0 of 78.8 years for both sexes combined, the e_0 of U.S. population in 2010. Life tables calculated based on a fraction of 2010 death rates imply an e_0 of 85.3 years; for a 0.50 fraction, 82.0 years; for a 0.75 fraction, 76.5 years for a 1.25 fraction; and 74.3 years for a 1.50 fraction. We create four life tables with e_0 ranging from -4.5 and -2.2 years to +3.2 and +6.5 years around the life table with e_0 of 78.8 years. These life tables are created by varying only the intercept value of the two-parameter Brass model life table, which raises or lowers survival at all ages relative to the standard life table but does not alter the slope of the standard life table. The e_0 for males and females and corresponding intercept values used in the Brass life tables are given in table 2.

Table 2: Intercept values of Brass life tables

Fraction of 2010 Age-specific Death Rates	e_0 for Both Sexes Combined	Males		Females	
		e_0	intercept	e_0	intercept
0.50	85.3	82.9	+0.20	87.7	+0.24
0.75	82.0	79.6	+0.08	84.4	+0.10
1.00	78.8	76.4	-0.04	81.2	-0.04
1.25	76.5	74.2	-0.24	79.0	-0.26
1.50	74.3	71.9	-0.46	76.7	-0.52

Source: Author

For the study of variations in life expectancy at age 65, we use the same five life tables discussed above for mortality after age 65, holding mortality below age 65 at the same level used for a life table with the medium level of $e_0 = 78.8$.

Immigration

The number of net immigrants is set 1.0 million per year in the baseline. The simulations range from 0.50, to 0.75, 1.25, and 1.50 times 1.0 million – or 500,000, 750,000, 1,250,000, and 1,500,000 per year respectively. A zero-immigration assumption is also introduced to examine stationary and stable populations. The U.S. Census Bureau has collected census-type data in recent years through American Community Survey. We use data from this survey to obtain age and sex information on recently arrived immigrants.

Demographic Effects

Variation in Fertility

The column (3) of Table 3 – labelled 1.00 for the fraction of 2010 age-specific fertility rates, which implies a total fertility rate of 2.00 – shows the rate of return if birth and death rates are the same as in 2010 and the annual net immigration is 1.00 million. The 1990-1994 birth cohort will have a 0 percent rate of return in their contributions. This is a birth cohort that will contribute to the pension scheme between 2010 and 2055. Later birth cohorts up to the latter part of the 21st century will have progressively larger negative returns.

The baby boom explains these negative returns. Larger cohorts share payments for the old among more people, so individuals pay less than if the cohort is small. If contributors receive a given subsequent pension regardless of their numbers (that is, if the tax rates are subsequently raised proportionally as the number of payers decreases), then larger cohorts gain and smaller cohorts suffer.

Table 3: Rate of return on unfunded pension contributions for successive birth cohorts, for five levels of fertility.

Birth cohort	Fraction of 2010 age-specific fertility rates				
	0.50	0.75	1.00	1.25	1.50
	Rate of return (percent)				
	(1)	(2)	(3)	(4)	(5)
1990-1994	-0.24	-0.17	0.00	-0.05	-0.03
2000-2004	-0.48	-0.33	-0.14	-0.11	-0.02
2010-2014	-0.72	-0.44	-0.21	-0.06	0.09
2020-2024	-0.88	-0.52	-0.23	0.01	0.21
2030-2034	-0.98	-0.58	-0.25	0.05	0.29
2040-2044	-1.02	-0.61	-0.26	0.07	0.35
2050-2054	-1.01	-0.61	-0.27	0.08	0.37
2060-2064	-0.95	-0.60	-0.27	0.07	0.36
2070-2074	-0.90	-0.59	-0.27	0.07	0.36
2080-2084	-0.88	-0.59	-0.28	0.07	0.36
Implied TFR	1.00	1.50	2.00	2.50	3.00

Source: Author

Other columns of Table 3 show the effect of lower or higher birth rates relative to that prevailed in 2010. If future birth rate decreases to half of the birth rate in 2010, then the negative annual rates of return increase to over 1 percent (column 1) by 2040-44. The increase in the negative rate of return will be less if future birth rate decreases to 0.75 times the birth rate in 2010. On the other hand, a 25 percent rise in the birth rate, which implies a total fertility rate of 2.50 and an annual rate of natural increase of about 0.5 percent, would provide all 2020-2024 and later birth cohorts with positive rate of return (column 4). A 50 percent increase in the birth rate, which implies a total fertility rate of 3.0 will lead to an increase in the annual rate of natural increase of slightly over 1 percent. This would lead to the rate of return of more than 0.3 percent for 2010-2014 cohorts (column 5).

Variation in Mortality

In contrast to the noticeable effect of variation in fertility, variation in mortality makes less of a difference. Table 4 shows negative returns for all birth cohorts for all levels of mortality, except for death rates that are one-half 2010 levels (implying a life expectancy at birth of 73.8 years, or 5.0 years less than the 2010 level). The 2040-2044 cohort would have a return of -0.01 percent if mortality is one-half of 2010 level, and -0.59 percent if mortality is 50 percent higher than~~t~~ 2010 level. The differences in mortality underlying these results are very large: the former corresponds to an expectation of life at birth of 85.3 years, and the latter to only 74.3 years. Recent annual life expectancy gains have averaged 1.9 per decade, and a reduction in mortality by one-half roughly corresponds to mortality improvements that might be expected over the next 35 years. One might be puzzled that such large differences in mortality rates have relatively smaller effects. In fact, mortality changes/differences produce offsetting effects: the contributor who suffers because their elders live longer also benefits themselves by drawing a pension for a longer period.

Table 4. Rate of return on unfunded pension contributions for successive birth cohorts for five levels of mortality, United States, fixed pension

Birth cohort	Fraction of 2010 age-specific death rates				
	0.50	0.75	1.00	1.25	1.50
	Rate of return (percent)				
	(1)	(2)	(3)	(4)	(5)
1990-1994	0.16	0.08	0.00	-0.12	-0.25
2000-2004	0.06	-0.04	-0.14	-0.29	-0.45
2010-2014	0.02	-0.10	-0.21	-0.38	-0.57
2020-2024	0.01	-0.11	-0.23	-0.42	-0.63
2030-2034	0.00	-0.12	-0.25	-0.45	-0.67
2040-2044	-0.01	-0.14	-0.26	-0.47	-0.70
2050-2054	-0.01	-0.14	-0.27	-0.47	-0.71
2060-2064	-0.01	-0.14	-0.27	-0.48	-0.72
2070-2074	-0.01	-0.14	-0.27	-0.48	-0.73
2080-2084	-0.01	-0.15	-0.28	-0.49	-0.74
Implied e_0	85.3	82.0	78.8	76.5	74.3

Source: Author

One might also be surprised that these differences are all in a consistent direction: returns are better when mortality is lower. While more favourable mortality increases the cost to those currently paying from the moment when mortality improves, it also increases even more the total return that they will eventually receive. Moreover, lower mortality, other conditions being equal, creates more rapid population growth which helps unfunded pension programs.

What about mortality at the oldest ages, supposing that mortality at ages under 65 years does not change? One may think that mortality changes only for the old population would make a great difference. As shown in Table 5, however, in old age mortality alone are less influential than variation in mortality at all ages. The difference between an expectation of life at age 65 of 22.9 years and of 16.9 (the difference between the first and last columns of Table 5) leads to rate of return ranging from -0.10 to -0.65 percent for the 2080-2084 birth cohort and even less for earlier cohorts.

These results suggest that enabling people to live longer does not make unfunded pension schemes greatly worse. Rather, a much greater problem is a declining birth rate. If people live longer after age 65, it means that contributors will need to pay more, but they will be eventually compensated by obtaining more when they become old.

Table 5: Rate of return on unfunded pension contributions for successive birth cohorts, for five levels of mortality above 65 years of age, U.S. population, fixed pension

Birth cohort	Fraction of 2010 age-specific death rates at age 65 and above				
	0.50	0.75	1.00	1.25	1.50
	Rate of return (percent)				
	(1)	(2)	(3)	(4)	(5)
1990-1994	0.12	0.06	0.00	-0.09	-0.20
2000-2004	0.00	-0.07	-0.14	-0.26	-0.39
2010-2014	-0.05	-0.13	-0.21	-0.34	-0.50
2020-2024	-0.07	-0.15	-0.23	-0.37	-0.55
2030-2034	-0.08	-0.16	-0.25	-0.40	-0.59
2040-2044	-0.09	-0.17	-0.26	-0.42	-0.62
2050-2054	-0.09	-0.18	-0.27	-0.42	-0.63
2060-2064	-0.10	-0.18	-0.27	-0.43	-0.64
2070-2074	-0.10	-0.18	-0.27	-0.43	-0.65
2080-2084	-0.10	-0.18	-0.28	-0.44	-0.65
Implied e_{65}	22.9	20.8	19.0	18.0	16.9

Source: Author

Variation in Net Immigration

Net immigration also makes only a small difference to rate of return (Table 6). With annual net immigration of 0.5 million (one-half of the baseline level), the rate of return to the 1990-1994 cohort are -0.13 percent, compared to -0.07 if there were 1.5 million net immigrants, or four times the base level. The effect is not strong enough that, by itself, would recommend fewer or more immigrants. The effects of immigration, when viewed over a lifetime, are modest because immigrants not only contribute over their working years but also subsequently draw benefits when they are old. Increased immigration has only a temporary benefit for the initial years after arrival, while the longer-term generational effect on returns is modest. Differences in the effect of immigration diminish slightly for later birth cohorts, with modest differences in the rate of return for the 2080-84 birth cohort.

Further Comparisons of Fertility and Immigration

Table 7 shows the same range of net immigration as in Table 6 but calculates the rate of return based on a fixed absolute number of 4.2 million annual births instead of using fixed age-specific birth rates. This produces immediate birth stationarity, in comparison to a stable age distribution that would be obtained by fixed age-specific birth rates only after the cohorts who were born before the commencement of fixed rates have died. The effect of a fixed number of births is obvious. For all immigration assumptions except zero immigration, the effects become increasingly negative. With heavy immigration of 1.5 million (shown in column (5)), along with fixed annual births of 4.2 million, the annual rate of return eventually stabilizes at –0.39 percent. If there were modest immigration of 0.5 million, the annual rate of return stabilizes at -0.34 percent and as shown in the first column, assumption of zero net immigration implies a long-term rate of return of 0 percent.

Table 6: Rate of return on unfunded pension contributions for successive birth cohorts, for five levels of immigration, U.S. population, fixed pension

Birth cohort	Fraction of 1,000,000 immigrants per year				
	0.50	0.75	1.00	1.25	1.50
	Rate of return (percent)				
	(1)	(2)	(3)	(4)	(5)
1990-1994	-0.13	-0.12	0.00	-0.09	-0.07
2000-2004	-0.25	-0.23	-0.14	-0.19	-0.18
2010-2014	-0.27	-0.25	-0.21	-0.21	-0.20
2020-2024	-0.27	-0.25	-0.23	-0.22	-0.20
2030-2034	-0.27	-0.26	-0.25	-0.23	-0.21
2040-2044	-0.28	-0.26	-0.26	-0.24	-0.22
2050-2054	-0.28	-0.26	-0.27	-0.24	-0.23
2060-2064	-0.28	-0.26	-0.27	-0.24	-0.23
2070-2074	-0.28	-0.26	-0.27	-0.24	-0.23
2080-2084	-0.28	-0.26	-0.28	-0.24	-0.23
Immigration (000)	500	750	1,000	1,250	1,500

Source: Author

Table 8 is similar to Table 7, but it varies the number of births rather than the number of immigrants. Comparing Tables 8 and 7 shows how much difference is due to the effect of births. Initially, the effect of births is greater than immigration for earlier birth cohorts. After the rate of return stabilizes, however, there is only a modest difference in the rate of return for either births or immigration.

Table 7: Rate of return on unfunded pension contributions for successive birth cohorts, for five levels of immigration and births fixed at 4.2 million per year, 2010 U.S. population, fixed pension.

Birth Cohort	Fraction of 1,000,000 Immigrants per Year				
	0.50	0.75	1.00	1.25	1.50
	Rate of return (percent)				
	(1)	(2)	(3)	(4)	(5)
1990-1994	-0.03	-0.02	0.00	0.02	0.03
2000-2004	-0.18	-0.16	-0.15	-0.13	-0.11
2010-2014	-0.26	-0.24	-0.23	-0.21	-0.19
2020-2024	-0.28	-0.27	-0.26	-0.25	-0.24
2030-2034	-0.30	-0.30	-0.30	-0.29	-0.29
2040-2044	-0.32	-0.32	-0.33	-0.33	-0.33
2050-2054	-0.32	-0.33	-0.34	-0.35	-0.36
2060-2064	-0.33	-0.34	-0.36	-0.37	-0.38
2070-2074	-0.33	-0.35	-0.36	-0.38	-0.39
2080-2084	-0.34	-0.35	-0.37	-0.38	-0.39
Immigration (1,000s)	500	750	1,000	1,250	1,500

Source: Author

Table 8: Rate of return on unfunded pension contributions for successive birth cohorts, for five levels of births with immigration of 1000000 persons per year, U.S. population, fixed pension

Birth Cohort	Thousands of Annual Births				
	3,200	3,700	4,200	4,700	5,200
	Rate of return (percent)				
	(1)	(2)	(3)	(4)	(5)
1990-1994	-0.04	-0.02	0.00	0.02	0.03
2000-2004	-0.23	-0.18	-0.15	-0.11	-0.08
2010-2014	-0.36	-0.29	-0.23	-0.17	-0.11
2020-2024	-0.45	-0.35	-0.26	-0.18	-0.11
2030-2034	-0.50	-0.39	-0.30	-0.21	-0.13
2040-2044	-0.51	-0.41	-0.33	-0.25	-0.18
2050-2054	-0.49	-0.41	-0.34	-0.28	-0.23
2060-2064	-0.45	-0.40	-0.36	-0.32	-0.29
2070-2074	-0.42	-0.39	-0.36	-0.34	-0.33
2080-2084	-0.40	-0.38	-0.37	-0.35	-0.34

Source: Author

Stationary and Stable Populations

Column (1) of Table 8 – with 500,000 net immigrants, the 2010 life table, and 4.2 million births each year – has special interest because it almost produces a stationary population. If mortality rates and replacement-level fertility is fixed and immigration is zero then it leads to a stationary population, the exchange feature of a pay-as-you-go pension scheme becomes clearer because there is no way for it to be either negative or positive in the long run – assuming only demographic factors. Thus, the rate of return stabilizes (as expected) at a level close to zero for a stationary population.

Table 9 shows the effect of stationarity in its pure form. It presents results for zero immigration, 2010 mortality, and different levels of a fixed annual number of births that lead ultimately to stationary population. As stationarity implies, the rate of return ultimately becomes zero irrespective of the level of fertility, starting with the cohort of 2050-2054, which is the first cohort to enter the pension scheme after the last unstable cohort passes through retirement.

Table 9: Rate of return on unfunded pension contributions for successive birth cohorts, for five levels of births, with zero immigration that leads to a stationary population, 2010 U.S. population, fixed pension

Birth Cohort	Thousands of Annual Births				
	3,200	3,700	4,200	4,700	5,200
	Rate of return (percent)				
	(1)	(2)	(3)	(4)	(5)
1990-1994	-0.12	-0.09	-0.07	-0.05	-0.03
2000-2004	-0.10	-0.08	-0.06	-0.04	-0.03
2010-2014	-0.08	-0.07	-0.05	-0.03	-0.02
2020-2024	-0.05	-0.04	-0.03	-0.02	-0.01
2030-2034	-0.03	-0.03	-0.02	-0.01	-0.01
2040-2044	-0.02	-0.01	-0.01	-0.01	0.00
2050-2054	0.00	0.00	0.00	0.00	0.00
2060-2064	0.00	0.00	0.00	0.00	0.00
2070-2074	0.00	0.00	0.00	0.00	0.00
2080-2084	0.00	0.00	0.00	0.00	0.00
Ultimate stationary population size (million)	258.8	299.0	339.2	379.4	419.6

Source: Author

Table 10 repeats in its zero percent column the column (3) of Table 9. Table 10 shows calculations based on the assumption that absolute number of births increases exponentially, ranging from annual rate of -2 and -1 percent to +1 and +2 percent. This is a quick way to generate a stable population from an arbitrary initial age distribution. In Table 10, the annual rate of change in births is reproduced in the rate of interest by participants in the scheme after 85 years, when the initial birth cohort has passed away. After about 85 years, the implicit rate of return is the same as the assumed annual rate of change in the number of births.

The general proposition regarding Table 10 is that a population increasing at *r* percent per year and arraying its pensions on a pay-as-you-go basis, will return to its participants an effective rate of real interest of *r* percent. Table 9 shows results for $r=0$ and table 10 illustrates results for a range of values of *r*. The formal statement for the expression for the premium of a scheme at rate of interest *r* is

$$\int_{\beta}^{\omega} e^{-rx} l(x)dx \Big/ \int_{\alpha}^{\beta} e^{-rx} l(x)dx$$

and this is identical to the ratio of the population over age β to that from α to β if the stable growth rate is *r*, as noted earlier in equation (1).

The results in Table 10 provide an explanation for the decrease in the rate of return shown in Figure 1. Higher fertility levels associated with a growing population produce higher rates of return. As fertility decreased in the U.S. population, population growth declined and the rates of return diminished. With relatively low fertility and moderate immigration, the implicit rates of return will remain negative.

Table 10: Rate of return on unfunded pension contributions for successive birth cohorts, with number of births Increasing from 4.2 million in 2010 at five different rates, with zero immigration that leads to stable populations, U.S. population, fixed pension

Birth Cohort	Annual increase (percent) in births from 4.2 million in 2010				
	-2	-1	0	+1	+2
	Rate of return (percent)				
	(1)	(2)	(3)	(4)	(5)
1990-1994	-0.09	-0.08	-0.07	-0.06	-0.05
2000-2004	-0.47	-0.34	-0.06	-0.03	-0.02
2010-2014	-0.71	-0.48	-0.05	0.01	0.03
2020-2024	-0.95	-0.58	-0.03	0.14	0.28
2030-2034	-1.23	-0.69	-0.02	0.34	0.68
2040-2044	-1.51	-0.80	-0.01	0.56	1.11
2050-2054	-1.74	-0.89	0.00	0.77	1.54
2060-2064	-1.89	-0.96	0.00	0.92	1.83
2070-2074	-1.97	-0.99	0.00	0.98	1.96
2080-2084	-2.00	-1.00	0.00	1.00	2.00

Source: Author

Discussion and Conclusion

This paper uses the metric of implicit rate of interest to study unfunded pensions under various demographic conditions. We make no attempt to examine the effect of economic growth. A growing economy helps a pension scheme as much as does a growing population. In the calculations here, we take people rather than the goods and services supplied to people, as our unit for study, and calculations are limited to demographic factors. The proper interpretation for these results is that they represent pure demographic effects, which would be superimposed on economic growth.

This study analyses effects of fertility, mortality, and immigration on the returns of an unfunded pension scheme. It illustrates the marked extent to which future fertility is more likely to be important for the rate of return that individuals will realise on their premiums than either mortality or immigration. There are several topics of interest that are not discussed in this paper. Two other effects that are worth studying are variation in labour force participation and age at retirement. This paper focusses solely on given benefits, which is the most common form of pay-as-you-go pension schemes. It would be useful to examine a fixed contribution approach in future research.

We also note a basic point concerning the equity between generations. Pay-as-you-go schemes not only redistribute income over a cohort's lifetime but also, in most cases by design, from better-off to the poor people. They also redistribute it – sometimes in a less equitable fashion – between generations. Keyfitz (1980) presents a cogent demographic critique of unfunded pension systems based on intergenerational equity issues, as well as recommending better public pension alternatives. In a fixed benefit scheme, income is redistributed from the members of smaller cohorts to the members of larger cohorts. In a fixed contribution scheme, the redistribution goes in the opposite direction. With either type of scheme, some cohorts will experience negative rates of return on their contributions.

Under the fixed benefit scheme that exists in the United States, most of those born before 2000 are likely to experience positive rates of return, as calculated from a purely demographic basis, but cohorts born after 2000 will probably experience negative rates. The rise of premium needed as the baby boom birth cohorts enter retirement ages, together with the prospect of negative returns, will put considerable additional stress on the public pension scheme. The present underlying challenges of such schemes, however, are not necessarily from demographic factors alone but also from the fact that the public unrealistically expects the relatively high returns to continue without change and the lack of political will to make/bring changes in existing public pension systems.

For developing countries such as India, the general implications of this paper are similar to those of the United States - pay-as-you-go unfunded pension schemes have inherent equity issues because the size of birth cohorts vary over time. Unlike the United States, however, where public pension covers most residents and has been in place for more than 85 years, public pension systems in many developing countries do not often cover all residents and are either new or, in some countries, do not yet exist. This means that changes to the public pension systems in developing countries may not involve such massive changes, as are required for reforms in U.S. public pension system. Keyfitz (1980) provides a useful discussion of public policy options for reforming unfunded pension systems. Although, Keyfitz (1980) focuses on the U.S. system, his discussion considers options for reform that are relevant for the study of unfunded public pension systems in developing countries.

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